

Simulation of Bearing Degradation by the Use of the Gamma Stochastic Process

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An effective predictive maintenance reposed on modeling, simulation, and on supervisory and prognostic techniques used to model the various phenomena. On this basis, and based on significant knowledge and parameters, we propose an approach based on stochastic processes that represent a mathematical structure for simulation, mainly the processes of continuous degradation and more particularly the Gamma process. Our work is devoted to the monitoring of the degradation process of the bearings at the level of a motor pump and makes it possible to evaluate the limiting operating time, as well as the evolution in time of the change of state. This methodology allows us to develop a mathematical model that describes the process of bearing degradation, thus providing a good prediction of failures and efficient maintenance planning for systems whose behavior is only partially predictable.

Keywords: Gamma law, predictive maintenance, vibration analysis, stochastic process, Gamma process, simulation.

1. Introduction

A stochastic process is a mathematical structure used to model the various phenomena where the prediction of the future is impossible for different reasons. Nowadays, with the increased reliability of mechanical components, it is increasingly difficult to estimate reliability from tests that take into account only the failure. Many components degrade before failure, these degradation measures contain very valuable

information about the reliability of the product, so he is considered that the failure occurs when the degradation reaches a certain limit [1].

Among the innumerable applications we cite in the industrial field: the reliability of the materials, the analysis of the measurement results and their planning, the prediction [2], so we can study the phenomena of degradation by the stochastic processes which have been the object of many articles in the last few decades especially the models of continuous degradation; the most frequently studied models are the Wiener process [3] and the Poisson process, as well as the Gamma process.

That's why this article will be used to define the gamma process and their application in reliability and predictive maintenance.

2. Various uses of the Gamma process

The gamma process is a continuous-state and positive-growth process. It allows to model the mechanisms of aging which are by nature slow, continuous and progressive.

Abdel-Hameed [4] was the first to propose the gamma process to model the degradation of a system, this use as a model of degradation is justified in [5] its properties allow its adaptation to many contexts: it is Adapted to the modeling of the propagation phase of any monotonous degradation and growing over time such as corrosion, crack propagation or erosion. The gamma process is used for the modeling of the phases of repairable systems in relation with their maintenance, so it is applied to research.

The conditional maintenance policy for a system subject to periodic inspections [6], and also for a system subject to a periodic inspection [7]. It is also used in sequential maintenance planning [8].

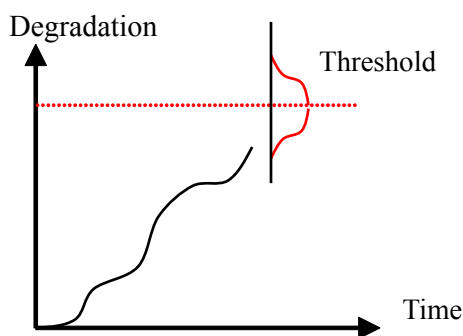


Figure 1 The process of degradation as a function of time

3. Stochastic model of a degradation process

The problem is to develop a stochastic model of the degradation process as a function of time that can be transformed into a model of predictive reliability, offline (before use) or online (in use after observing the level of degradation), look at Fig. 1.

Thus, the aim of this work is to define the process of degradation of the results obtained in a study on the follow-up of a bearing installed on a motor pump at the Skikda refinery, using the gamma process, which is a process Stochastic model to simulate the increasing degradation of an aging subject over time, as well as the estimation of reliability from degradation measurements [1].

4. Mathematical properties of Gamma process

Before discussing the gamma process, firstly it is necessary to remember some properties of the Gamma distribution as a basis for future analysis.

The gamma function equals:

$$\Gamma(x) = \int_0^{+\infty} u^{x-1} e^{-u} du \quad x > 0 \quad (1)$$

The Gamma distribution (law) $\Gamma(\alpha, \beta)$ has the probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-x\beta} \quad x > 0 \quad (2)$$

with $\alpha > 0$ shape parameter and $\beta > 0$ scale parameter. X follows a gamma distribution $\Gamma(\alpha, \beta)$, then the expectation and the variance are presented in Eq. (3) and Eq. (4):

$$E(x) = \frac{\alpha}{\beta} \quad (3)$$

$$var(x) = \frac{\alpha}{\beta^2} \quad (4)$$

The gamma process is a random process with positive, independent increments that can be stationary or non-stationary. It differs from the Wiener process by its trajectories *càd-làg* (continuous to the right, limit to the left) and by its positive increases.

The process $X(t)$ is a stationary Gamma process $Ga(\alpha, \beta)$ which translates linear degradations on average [9] if:

1. $X(0) = 0$,
2. $X(t)$ is a process with independent and positive increments,
3. for all $t > 0$ and $\Delta t > 0$, the increase law $X(t + \Delta t) - X(t)$ is a gamma distribution.

5. Data analysis

We used the vibration history (rms value) of the *rigid bearing 6316* with a single row of balls. Beginning with the execution of the statistical test on the vibration measurements to summarize the information on our sample $(\alpha, \beta, H_0, \alpha)$ with:

1. α – shape parameter,
2. β – scale parameter,

3. α – the degree of risk (confidence interval),
4. H_0 – hypothesis tested by the Kolmogorov fitting.

Then estimate the reliability of data and the theoretical one. Finally, simulation by Gamma process.

This study was carried out using Easyfit 5.4 statistical software. This software makes it possible to check whether the measurements follow a gamma law or not. And here below in Table 1 and Fig. 2 the results of measurements which represent the effective values of the evolution of the bearing degradation over time.

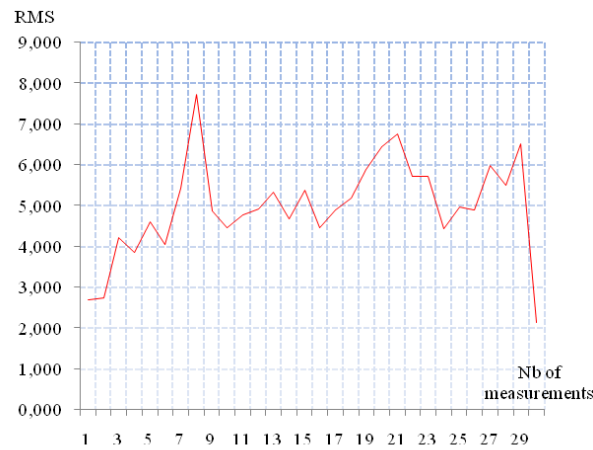


Figure 2 Evolution of bearing degradation

The Kolmogorov-Smirnov fitting allows testing the hypothesis H_0 according to which the observed data are generated by a theoretical probability law considered as a suitable model and here is the result of the test:

Sample size 30, follows a Gamma law with:

- statistic = 0.15253
- $p = 0.44439$

Therefore according to Gamma the vibration measurements follow a gamma law with a parameter of form $\alpha = 17.36$ and a parameter of scale $\beta = 0.28$.

The graphic representation below Fig. 3 and Fig. 4 shows that the data are very close to that of the gamma distribution.

The analysis as well as the calculation of the reliability in the field of the mechanics is very important tools to characterize the behavior of the product in the different phases of life; Measure the impact of design changes on product integrity,

Table 1 Vibration measurements

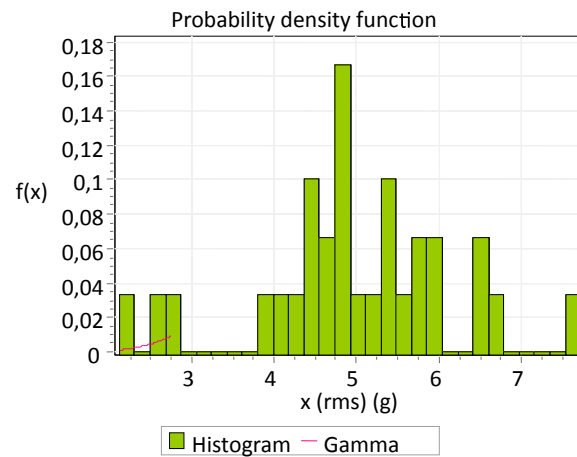
Measure Number	RMS value (g)	Measure Number	RMS value (g)	Measure Number	RMS value (g)
1	2.678	11	4.770	21	6.748
2	2.743	12	4.915	22	5.714
3	4.213	13	5.316	23	5.712
4	3.839	14	4.667	24	4.438
5	4.589	15	5.384	25	4.956
6	4.029	16	4.448	26	4.882
7	5.414	17	4.882	27	5.971
8	7.723	18	5.175	28	5.489
9	4.856	19	5.887	29	6.508
10	4.453	20	6.436	30	2.126

Table 2 Fitting results

a	0.2	0.1	0.05	0,02	0.01
Critical value	0.19032	0.21756	0.2417	0.27023	0.28987
To reject?	No	No	No	No	No

qualify a new product and improve performance throughout its mission, Reliability expressed by the function $R(t)$ is given by:

$$R(t) = 1 - F(t) \quad (5)$$

**Figure 3** Probability density function according to the gamma distribution

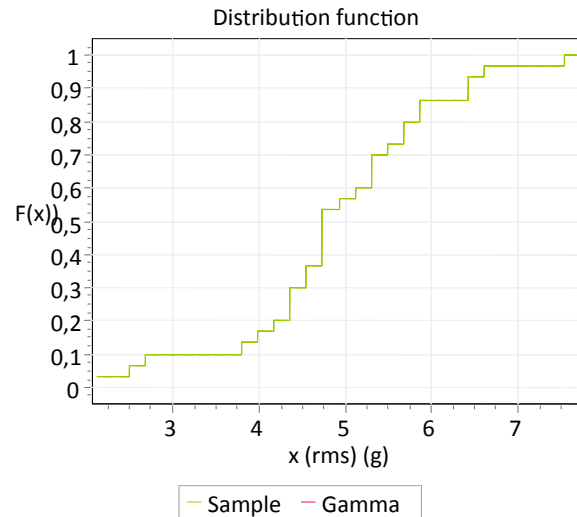


Figure 4 Distribution function according to the gamma distribution

In order to determine the values of $R(x)$, we can compute the distribution function of the variable x by the following formula: $F(x) = \text{GAMMA LAW}(x; \alpha; \beta; \text{True})$ after the calculation Fig. 5 represents the reliability function of the tests which follows the Gamma law.

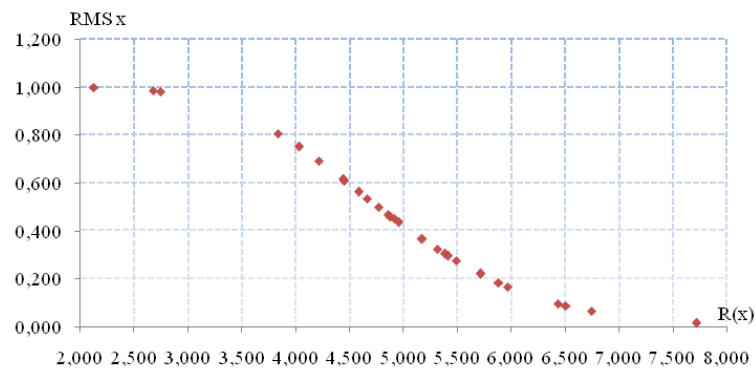


Figure 5 Reliability function of the tests

6. Simulation of the degradation process

In section 4, the Increment law $X(t + \Delta t) - X(t)$ is a gamma distribution and, according to the statistical test, the vibration measurements follow the gamma law

so that the degradation or evolution of Level of bearings by the gamma process. The simulation of the degradation trajectories is carried out according to a program under MATLAB; the results are presented in Fig. 6 and Fig. 7.

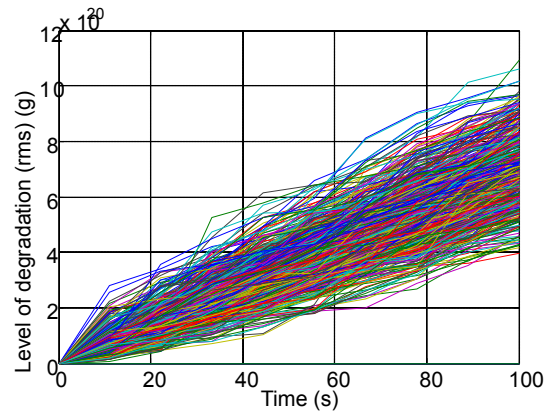


Figure 6 Plot of 500 trajectories according to a Gamma process parameters $\alpha = 17.36$ $\beta = 0.28$

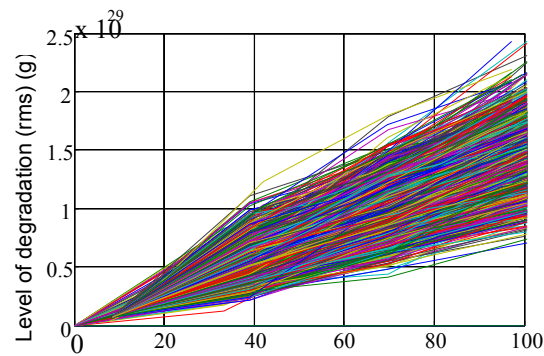


Figure 7 Plot of 2000 trajectories according to a Gamma process parameters $\alpha = 17.36$ $\beta = 0.28$

7. Interpretation of results

The performance of the statistical test shows that the vibration measurements of the bearing degradation follow a Gamma law $G(\alpha, \beta)$ with a scale parameter $\alpha = 17.36$ and a shape parameter $\beta = 0.28$, according the Kolmogorov fitting the observed data are accepted by the decision rule whatever is of $a = [0.2-0.1-0.05-0.02-0.01]$ so that the hypothesis H_0 has been realized, the values of α represent the values of the Confidence interval, the statistical test is less than the critical value so the decision rule is accepted.

Example: for a risk $a = 0.01$

- The statistical test is worth: $D = 0.15253$
- The critical value is: $D_0 = 0.28987$
- Decision rule: $D < D_0$ accept

All this allows us to identify the bearing behavior by using the Gamma stochastic process to minimize maintenance and consequently maintenance costs. We find a good match between the two curves of the reliability function that of the measured data with respect to the theoretical curve.

Track the progress of degradation of the bearings over time permit to model the phenomenon by the Gamma process, because degradation can be temporarily reduced by phenomena of improvement [9], the simulation by the Gamma process provides a good prediction. So this methodology allowed us to simulate and identify the phenomenon of the degradation of bearings by the Gamma process.

8. Conclusion

The state of degradation of an element is determined by the variations of a certain parameter of state X . If the value of the parameter X exceeds a certain threshold tolerated and fixed by the repair technology or the technical clauses of use, Repair becomes necessary because it is considered a failure. The variations of parameter X form a random process in time. In this work, we presented a statistical analysis methodology using the Gamma process to analyze test results and to verify the possibility to identify the behavior of the system and simulating the degradation of bearings, and grace to the vibratory study which highlighted the origin of the failure of the bearings.

The adjustment of a Gamma process from the test data allowed to estimate the reliability of the systems to even anticipate and avoid unexpected failures by a good planning of the operations of the maintenance.

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